Confidential

basic education

Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS P1

NOVEMBER 2024

MARKS: 150

TIME: 3 hours

This question paper consists of 9 pages and 1 information sheet.

MATHE1

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 12 questions.
2. Answer ALL the questions.
3. Number the answers correctly according to the numbering system used in this question paper.
4. Clearly show ALL calculations, diagrams, graphs, etc. that you have used in determining your answers.
5. Answers only will NOT necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round off answers to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. An information sheet with formulae is included at the end of the question paper.
10. Write neatly and legibly.

QUESTION 1

1.1 Solve for x:

1.1.1 $x(x-3)=0$ (2)

1.1.2 $2x^2+1=4x$ (correct to TWO decimal places) (4)

1.1.3 $x^2-2x-3>0$ (4)

1.1.4 $2^{2x}-2^{x+2}-32=0$ (5)

1.1.5 $\sqrt{-2x+4}-x=2$ (4)

1.2 Solve for x and y simultaneously:

$2x+y=3$
$y^2+xy=2$ (5)

1.3 Consider the product $\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\left(1+\frac{1}{4}\right)\ldots$

Each factor in the product is of the form $\left(1+\frac{1}{n}\right)$ for $n\geq 2$.

Determine ALL the values of n for which the product will be an integer value. (3)
[27]

QUESTION 2

2.1 The first term of an arithmetic series is 7. The common difference of this series is 5 and the series contains 20 terms.

2.1.1 Calculate the sum of this series. (2)

2.1.2 The original arithmetic series is extended to 75 terms. The sum of these 75 terms is 14 400. Using sigma notation, write down an equation for the sum of the terms added to the original series. (4)

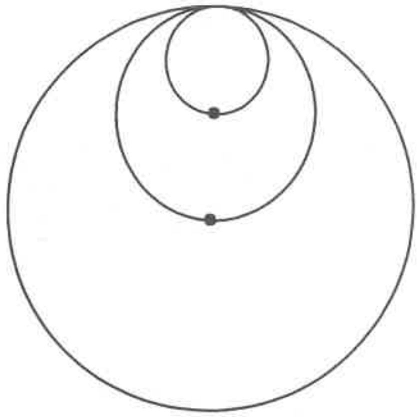
2.2 The sequence of the first differences of a quadratic pattern is: 1 ; 3 ; 5 ; …

2.2.1 If T_{99} of this quadratic pattern is 9 632, calculate the value of T_{98}. (3)

2.2.2 If it is further given that the third term of the quadratic pattern is 32, determine the general term, T_n, of the quadratic pattern. (5)
[14]

QUESTION 3

A circle with radius 6 cm is drawn.
A second, smaller circle is drawn through the centre of the first circle and also touches the first circle internally, as shown in the diagram.
A third, smaller circle is drawn through the centre of the second circle and touches the second circle internally. The process of drawing circles continues and forms a geometric pattern.



3.1 Write down the radius of the 3^{rd} circle. (2)

3.2 Calculate the sum of the areas of the first 10 circles. (4)

3.3 Which circle has a diameter of $\frac{3}{128}$ cm? (4)
[10]

QUESTION 4

Given: $f(x) = a^x - 1$ for $a > 0$. $B\left(2\,;\frac{-5}{9}\right)$ is a point on f.

4.1 Calculate the value of a. (2)

4.2 Write down the range of f. (1)

4.3 Sketch the graph of f. Clearly show the intercepts with the axes and asymptotes, if any. (3)

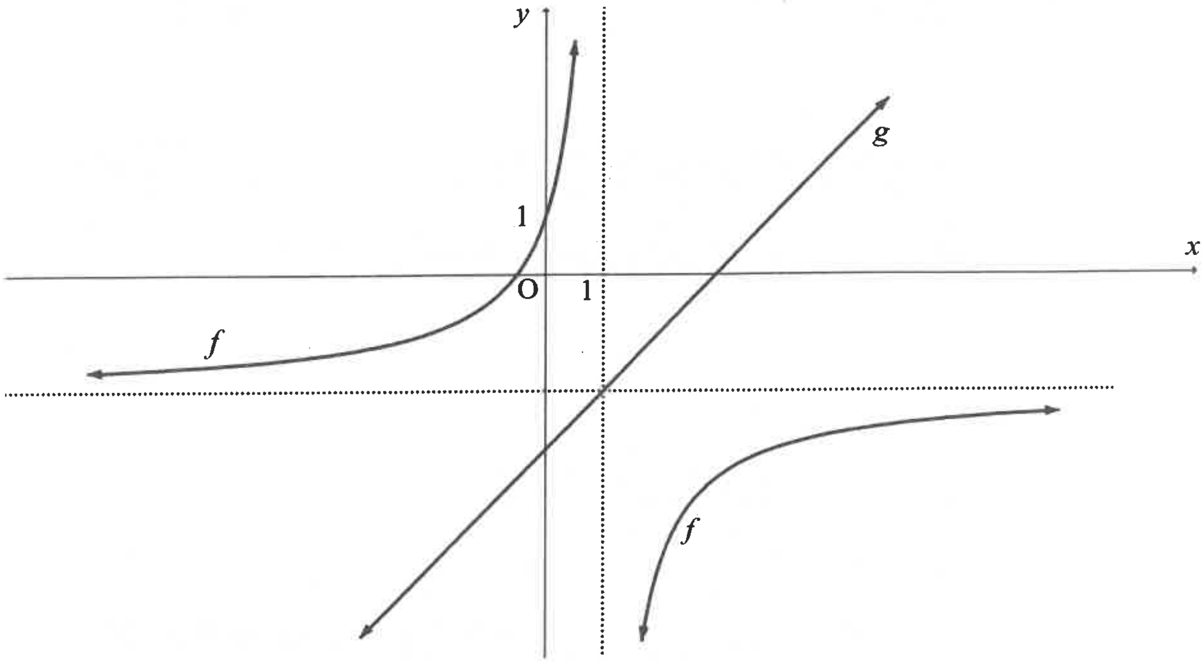
4.4 It is further given that C is a point on f at $y = \frac{19}{8}$.

Determine the coordinates of C', the image of C, when C is reflected about the line $y = x$. (3)
[9]

QUESTION 5

Sketched below is the graph of $f(x)=\dfrac{a}{x+p}+q$ having the domain $(-\infty\,;1)\cup(1\,;\infty)$.

The graph of f cuts the y-axis at $(0\,;1)$. A line of symmetry of f is given by $g(x)=x-3$.



5.1 Write down the value of p. (1)

5.2 Determine the equation of the horizontal asymptote of f. (2)

5.3 Calculate the value of a. (2)

5.4 For which values of x is $f(x)\geq 0$? (3)

5.5 Graph f undergoes a transformation to h where:

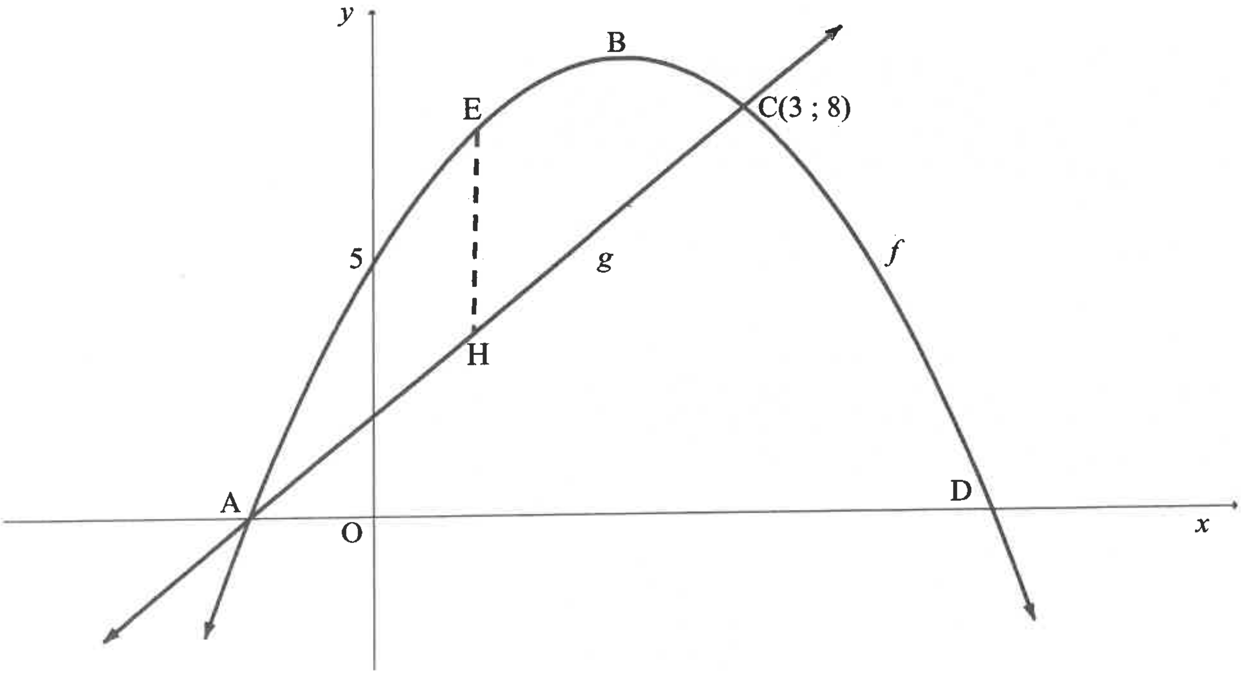
- The domain and range of h are the same as that of f
- $h'(x)$, the derivative of h, is negative on its domain

Describe a possible transformation that f could have undergone to result in h. (2)

[10]

QUESTION 6

In the diagram below, the graphs of $f(x) = -x^2 + 4x + 5$ and g, a straight line, are drawn. $C(3\,;8)$ is a point of intersection of f and g. EH is drawn parallel to the y-axis, with E a point on f and H a point on g.



6.1 Calculate the coordinates of B, the turning point of f. (3)

6.2 Show that the equation of the line through A and C is given by $g(x) = 2x + 2$. (3)

6.3 Calculate the maximum length of EH for $f > g$. (4)

6.4 Given: $k(x) = f(x+m) = -x^2 - 2mx - m^2 + 4x + 4m + 5$

Determine the value of m such that g is a tangent to k. (5)

[15]

QUESTION 7

7.1 Mary's grandparents deposited R5 000 into a savings account on the day that she was born. The account pays interest at a rate of 6,8% p.a., compounded quarterly. Calculate the accumulated amount in this account on Mary's 16th birthday. (3)

7.2 After 4 years, the value of a printer was half of its original value. Determine the rate at which the value of the printer depreciated over this period, if depreciation was calculated according to a straight-line method. (2)

7.3 Tshepo was granted a loan of R100 000 on 1 March 2022 at an interest rate of 13,5% p.a., compounded monthly. Tshepo agreed to repay the loan over 5 years in monthly instalments of R2 300,98, starting on 1 April 2022.

7.3.1 Calculate the total interest that he will pay over the 5 years. (2)

7.3.2 Tshepo paid R22 300,98 (his monthly instalment and an additional R20 000) on 1 March 2024 into the loan account. He continues to pay the original monthly instalment thereafter. How many months earlier will Tshepo repay the loan? (7)
[14]

QUESTION 8

8.1 Determine:

8.1.1 $\frac{d}{dx}\left[3x-5x^2\right]$ (2)

8.1.2 $g'(x)$ if $g(x)=\frac{2}{x^2}-\sqrt[3]{x^7}$ (4)

8.2 Determine the equation of the tangent to $f(x)=x^3-4x^2+2x+3$ at $x=2$. (3)

8.3 Given: $f(x)=-6x^2$

8.3.1 Determine $f'(x)$ from first principles. (5)

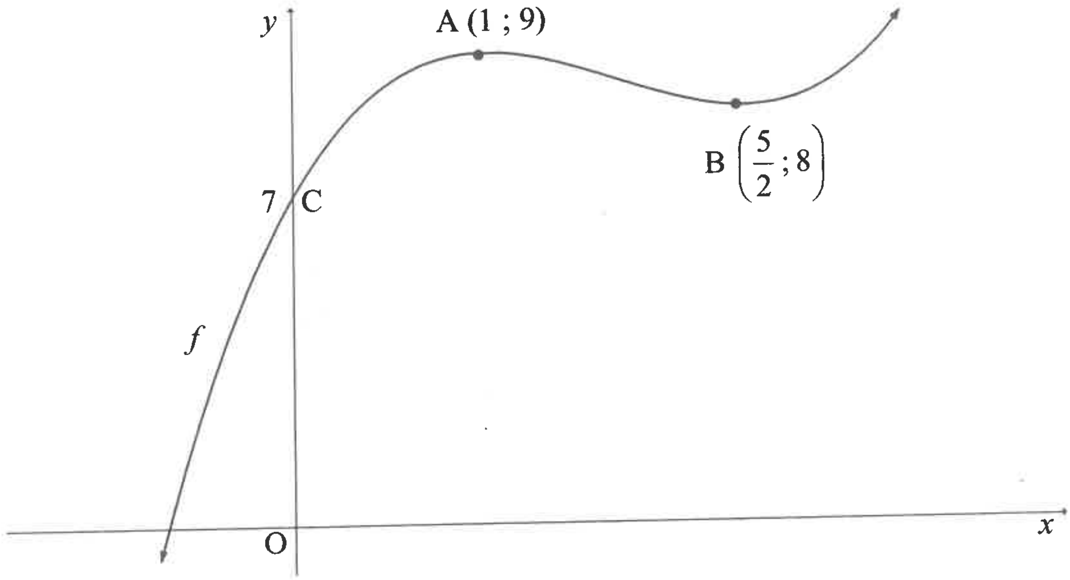
8.3.2 Write down how you will restrict the domain of f such that f^{-1}, the inverse of f, is a function. (1)

8.3.3 Determine the equation of f^{-1} for $f^{-1}(x)\le 0$. Write your answer in the form $y=\ldots$ (3)
[18]

QUESTION 9

A(1 ; 9) and $B\left(\frac{5}{2};8\right)$ are the turning points of graph f below.

C(0 ; 7) is the y-intercept of f.



9.1 For which values of x is f decreasing? (2)

9.2 Write down the x-intercepts of f', the derivative of f. (2)

9.3 For which values of x will f be concave up? (2)

9.4 Determine the value of k for which $y = f(x) + k$ will have THREE positive x-intercepts. (2)
[8]

QUESTION 10

A cyclist rode from town P and stopped at town T. The speed (in km/h) at which this cyclist rode, is represented by the equation $s'(t) = -3t^2 + 18t$.

NOTE: Speed is the rate of change in distance with respect to time.

10.1 Calculate the maximum speed that the cyclist reached on this ride. (3)

10.2 Calculate the distance between town P and town T. (5)
[8]

QUESTION 11

A certain number of learners are sitting for examinations in Mathematics, Tourism and Geography.

- All these learners sit for at least one of these examinations.
- The total number of learners who sit for Mathematics (M), is 22.
- The total number of learners sitting for Tourism (T), is 16.
- The total number of learners sitting for Geography (G), is 18.
- 5 learners sit for Mathematics and Tourism, but not Geography.
- 4 learners sit for Mathematics and Geography, but not Tourism.
- 3 learners sit for Tourism and Geography, but not Mathematics.
- 6 learners sit for only Tourism.

11.1 Draw a Venn diagram to represent ALL the learners sitting for these examinations. (3)

11.2 Calculate the probability that a learner, chosen at random, will sit for examinations in at least TWO of the subjects. (2)

11.3 Determine if the events: sitting for examinations in Mathematics and sitting for examinations in Tourism are independent. Support your answer with the necessary calculations. (4)
[9]

QUESTION 12

A company generates a 4-character code using the 26 letters of the alphabet and the 10 digits, from 0 to 9.

The code is in the form:

letter	digit	letter	digit

12.1 Determine how many different codes can be formed if letters and digits may be repeated. (2)

12.2 Determine how many different codes can be formed if:

- The letters D, F, I, Q, U and V may NOT be used
- The code may NOT start with a W or a Z
- Letters or digits may NOT be repeated
- The code ends with an odd digit (4)

12.3 The company wishes to increase the number of 4-character codes formed in QUESTION 12.2 by allowing the letters D, F, I, Q, U and V to be used. Calculate the percentage increase in the number of different codes that can now be formed. (2)
[8]

TOTAL: 150

INFORMATION SHEET

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$A = P(1+ni)$ $\quad A = P(1-ni)$ $\quad A = P(1-i)^n$ $\quad A = P(1+i)^n$

$T_n = a + (n-1)d$ $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$T_n = ar^{n-1}$ $\quad S_n = \frac{a\left(r^n - 1\right)}{r-1}\ ;r \neq 1$ $\quad S_\infty = \frac{a}{1-r}\ ; -1 < r < 1$

$F = \frac{x\left[(1+i)^n - 1\right]}{i}$ $\quad P = \frac{x\left[1-(1+i)^{-n}\right]}{i}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $\quad \mathrm{M}\left(\frac{x_1 + x_2}{2} ; \frac{y_1 + y_2}{2}\right)$

$y = mx + c$ $\quad y - y_1 = m(x - x_1)$ $\quad m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad m = \tan\theta$

$(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$$

$\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$ $\quad \sigma^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $\quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $\quad b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$